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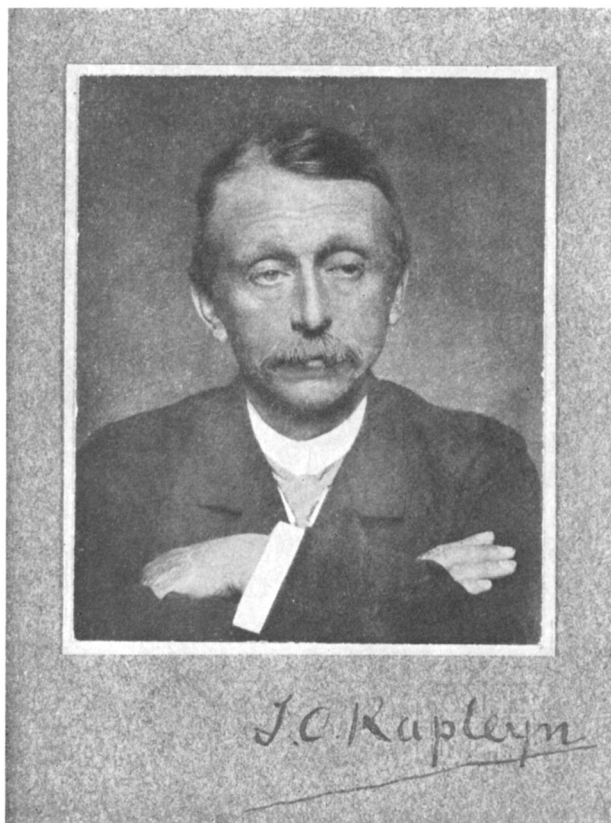
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**JACOBUS CORNELIUS KAPTEYN**  
**1851-1922**

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J. C. KAPTEYN

By FREDERICK H. SEARES

Jacobus Cornelius Kapteyn, one of the most distinguished astronomers of his generation, died at Amsterdam, on June 18, in his seventy-second year.

Kapteyn presented the unique figure of an astronomer without a telescope. More accurately, all the telescopes of the world were his. For long years he held the professorship of astronomy at the University of Groningen, which possessed no observatory of its own. Lacking funds to procure instruments that would yield data needed for his investigations, Kapteyn established at Groningen the famous astronomical laboratory that now bears his name. His genius, his rare skill in organizing large undertakings, and his kindly, winning personality, quickly gained the sympathetic cooperation of almost every important observatory throughout the world. A flood of observational data poured in upon him at Groningen, and in return there came from his laboratory a long series of published results, easily of first importance in their bearing on our knowledge of the structure and extent of the universe of stars.

That Kapteyn so easily won the interest and support of others for his own investigations was due much to his own ready spirit of cooperation, well illustrated by one of his earlier undertakings. During the years 1887 to 1891, Gill, Her Majesty's Astronomer at the Cape of Good Hope, had photographed the southern heavens for the purpose of constructing a catalogue of stars. The stars of the southern hemisphere were then little known, the fainter ones not at all. Gill's photographs showed images of nearly half a million of them; but he was unable to find funds for the measurement of the negatives and the laborious calculation and listing essential for the formation

of the catalogue. Officially the work was that of the British Government, but at this point Kapteyn came forward, volunteering to measure the photographs in his laboratory and prepare the catalogue for the printer, and to provide the money necessary for the completion of this part of the work. To this investigation he gave more than twelve years, without other return than the satisfaction of having done well an epoch-making piece of work. His resourcefulness in mastering material difficulties is illustrated by his appeal at a critical moment to the prison authorities of Holland for the assistance of certain convicts of exceptional intelligence to aid him in routine calculations. The request was granted. It is scarcely likely that on any other occasion a convict contributed to the cause of science.

Kapteyn's thoughts ever centered about the great problem of the structure of the universe. Inherently complex, the problem was made still more difficult by the lack of data upon which to base investigation, and he resolutely set himself the task of supplying this need, or rather, that of providing an organization by means of which the necessary information could be gathered, a labor far too great for any individual or even any single institution. Skilful planning was required, for the stars are far too numerous to be observed as a whole. At the same time, it was essential that objects in all parts of the sky be examined, for the characteristics of the stars vary, especially with distance from the Milky Way. Kapteyn therefore selected for intensive study about 200 comparatively small areas, uniformly distributed over the sky from pole to pole. These are known today to every astronomer simply as the Selected Areas. In addition, he outlined with great clearness and detail, in a small pamphlet<sup>1</sup> that has become famous, exactly what must be learned about the stars in these regions—their numbers, brightness, motions, distances, spectral types, etc.—before questions as to the size and form of the universe can be attacked successfully; and, further, he so fired with enthusiasm his colleagues all over the world that they began at once to accumulate the necessary information. The first fruits of this great undertaking have begun to ripen, but it will be many years before the harvest ends. The discus-

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<sup>1</sup>*Plan of Selected Areas*, Groningen, 1906.

sion of many important matters must now pass to other hands, but the work so well begun will be pushed to completion.

The underlying unity of Kapteyn's work does not at first meet the eye. Were one to consider simply the titles, and in many cases even the contents of Kapteyn's numerous papers, he might easily gain the impression that they represent investigations of unrelated questions. Superficially there is little connection between the many determinations of proper motion, the parallaxes of the helium stars, the mean parallaxes of stars as a function of proper motion and magnitude, the numbers of stars in each interval of magnitude, galactic latitude and spectral type, and the numbers of stars between specific limits of proper motion. But any inference that these investigations were unrelated in the mind of Kapteyn would be the farthest possible from the truth. Rarely have investigations been more carefully or thoughtfully planned than those undertaken by him, a fact well known to everyone who came into scientific relations with him and exemplified in concrete form in the pages of his "Plan of Selected Areas." And always they had very immediate bearing on the problem of the form and structure of the universe which was the guiding principle in all his work. Thus various aspects of proper motion reappear in his parallaxes, both individual and mean, and these, together with the results for the number of stars of different magnitudes, are the basis for the luminosity and density functions which determine the spatial distribution of stars, while his results for stellar distribution, as we shall see later, led him to an explanation of star streaming whose existence he had detected from a study of certain other aspects of proper motion. All this becomes clear to one who reads his contributions attentively, and to some extent it is stated explicitly in his papers, "On the Distribution of Stars in Space," and "Theory of the Arrangement and Motion of the Sidereal System";<sup>2</sup> but no one knew these inter-relations and their relative significance better than Kapteyn himself, and it would have made the signal importance of his work more con-

<sup>2</sup>*Mt. Wilson Contr.*, Nos. 138, 230; *Astrophysical Journal*, **52**, 23, 1920; **55**, 302, 1922.

spicuous had he himself summarized in written form the mutual dependence of the numerous questions that he studied.

An adequate discussion of Kapteyn's service to astronomy is beyond the scope of this article, but something of the value of that service may be illustrated by an account of one phase of his work—that relating to star streaming. As a matter of fact, the essential unity of his effort makes it impossible to isolate and discuss separately any single investigation. And so with stream motion: other investigations, incidentally at least, must also be considered; but in what follows emphasis will be thrown upon his treatment of this important and very striking phenomenon.

The greatest task involved in any attempt to trace the form of the stellar universe is the determination of the distances of large numbers of individual stars. Before the development of the spectroscopic method of measuring stellar parallax by Adams and his associates, such a determination, practically speaking, was impossible. The distances of all but the nearest stars are so great that the ordinary method of measuring parallax fails. It was necessary, therefore, to proceed indirectly, using methods which depend upon the statistical treatment of large masses of data. Important among these data are proper motions.

The grouping of stars into constellations is today essentially what it was 2000 years ago when the earliest catalogues were made; but we now know that underneath the illusion of permanence in the mutual distance of the stars, as we see them scattered over the face of the sky, minute changes constantly take place. The change in the direction in which a star is seen, expressed in angular units, in, say, a year or a century, is its proper motion. The existence of proper motions means simply that the individual stars, like individual bees in a swarm, are moving with respect to the center of the swarm. Judged by terrestrial standards, the velocities of the stars are very large, sometimes hundreds of kilometers per second, but the observed angular changes in position on the face of the sky in any moderate interval of time are, nevertheless, almost vanishingly small, because of the enormous distances separating the stars from the observer.

The question in which Kapteyn was primarily interested

therefore led him naturally to a careful examination of all the phenomena of proper motion, and it was through certain peculiarities affecting the motions of stars in different parts of the sky that star streaming was discovered. The observed displacements differed from those to be expected on the basis of motions wholly at random, and a study of these deviations led to the conclusion that the stars belong to one or the other of two interpenetrating swarms, moving relatively to each other.

The gradual development of the ideas that led to this discovery is of interest from the standpoint of scientific methodology, and, like other discoveries, for example, the variation of latitude, shows the disadvantage of attempting to apply deductive processes in the early stages of an investigation, especially when there is little ground for the choice of hypotheses to serve as premises. Thus dynamical considerations lead one to expect some sort of systematic motion among the stars, and various types of motion had been sought for. Schönfeld considered the possibility of a rotation of the Milky Way as a whole, but found nothing. Others supposed the stars to be moving with different periods in orbits lying in the plane of the Milky Way. Some assumed a common direction of motion, others tried a combination of direct and retrograde motions. Kapteyn himself looked for a preferential motion parallel to the Milky Way, its characteristics in other respects being arbitrary. None of these trials met with success. The method was faulty, for it was based on undirected guesses as to what the actual state of affairs might be; and, as Kapteyn subsequently remarked, "How long might we have guessed before we put the question, Are there two star streams?"

After much wasted effort, Kapteyn saw the inherent weakness and modified his attack, proceeding more inductively. Actually, however, his first step was an assumption, followed by deduction and a comparison of theoretical results with observation. He assumed the existence of random motion, that is, the absence of systematic motion of any form whatever, derived its consequences, and studied their relation to the observed motions. This at once dem-

onstrated the existence of systematic characteristics in the latter, and pointed the way (inductively) toward interpretation. The economic gain of this procedure lay in the circumstance that the presence of systematic motions among the stars was demonstrated directly, while the guesses at interpretation received a directional impulse from the data themselves. The range of possible explanations was thus very much narrowed, only two or three interpretations being admissible.

Turning now to details, let us consider first the consequences of a random distribution of stellar motions. By random motion we mean that there is no preferential direction; that in a large group of stars equal numbers move north, south, east, west, and in all other directions; or, stated in another way, that equal numbers of stars have motions lying between the directions N. and NE., NE. and E., E. and SE., etc. Actually, however, another element enters, for the proper motions of stars are not of equal magnitude; and in a random distribution there will be no special preference for motions of any given magnitude in any given direction.

To obtain a clearer picture of what this means, suppose the observer to be at rest near the center of the system, and let us make a diagram representing the motions of the stars as seen by him in any small region of the sky. The observed displacements are of course projections on the surface of the sky of the actual velocities, which, on the assumption of random motion, have all possible directions in space. Trace first (Fig. 1) the direction lines NS., EW., etc., N. being the direction toward the North Pole of the sky. Then from the intersection point draw lines representing in direction and magnitude the proper motions of each of the stars in the region in question. If the number of stars is large and if their motions are wholly random, the diagram will be very nearly symmetrical about the central point. Approximately the same number of lines of a given length will be found in each sector, and, even more approximately, the total number of lines in each sector of the diagram will be the same. The

approximation to symmetry will become more evident if we add together all the proper motions in each sector, divide by the number, and draw in the middle of the sector a single line representing this average proper motion, for the whole sector (Fig. 2). The extremities of these lines will lie very nearly on a circle whose center is at the intersection point. Random motion thus implies a radial symmetry in the distribution of the total proper motions of the stars, whatever the region of the sky examined.

The observer, however, is not at rest, for the solar system to which he belongs is moving through space at the rate of some 20 km per second, in what for the present we must

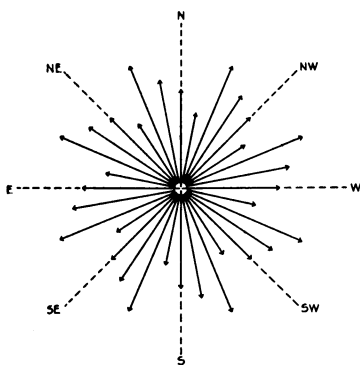


Fig. 1

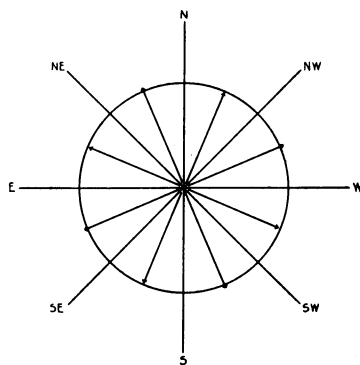


Fig. 2

consider to be a straight line, directed toward a point not far from the bright star *Vega*. The Earth of course is also in motion around the Sun, but in general it is only the motion of the system as a whole, which is cumulative in its effect, that produces any measurable influence. The progressive motion of the solar system also contributes to the change in direction in which any star is seen, giving rise to a component called the parallax displacement, while the change due to the star's own velocity is known as the peculiar motion. The parallax motion is an apparent drift to the rear, similar to the apparent motion of objects seen from the window of a moving train.

The observed distribution of the proper motions, even

when the velocities are at random, will not, therefore, be that shown in Figs. 1 and 2. To find the real diagram, the peculiar motion of each star must be compounded with its parallactic displacement by the ordinary parallelogram construction. The parallactic displacement depends upon the

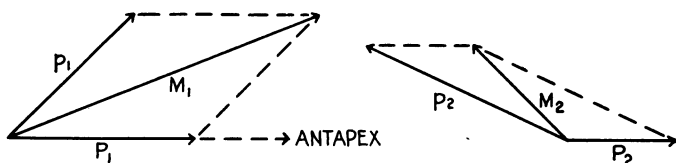


Fig. 3

distance of the star, being largest for the nearest stars, but is always directed toward the antapex, a point on the sky diametrically opposite to that toward which the solar system is moving (apex). Thus for any two stars, considered by themselves,  $p_1$  and  $p_2$  (Fig. 3) represent their peculiar motions, that is, what their proper motions would be were the observer at rest,  $P_1$ ,  $P_2$  their parallactic motions, and  $M_1$  and  $M_2$  the actually observed proper motions.

If this composition process be applied to each of the stars considered in Fig. 1, a moment's reflection will show that

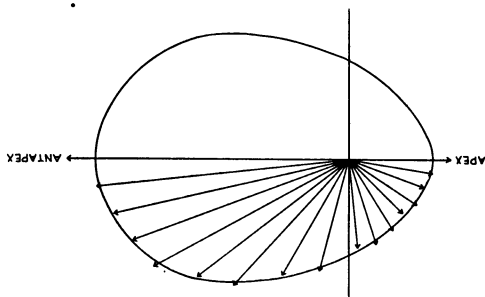


Fig. 4

Fig. 2 must be replaced by something like Fig. 4, in which the extremities of the lines, here representing the total proper motion in each sector, lie on an oval with the origin displaced from the center, and the major axis directed toward the apex and antapex of the observer's motion. It will

be noted, too, that the symmetry of Fig. 2 will be disturbed, not only in the matter of the amount of motion in each sector, but also in the number of proper motions in each. Consider a star having a small peculiar motion directed toward the apex. For an observer at rest this will be the observed direction of the motion; but for a moving observer the observed proper motion will be in the opposite direction, provided the parallactic displacement, which is always toward the antapex, exceeds the star's peculiar motion. The sector toward the antapex (Fig. 4) will therefore include more motions than that toward the apex. This is an important point, utilized to its full worth by Kapteyn.

If the solar motion be exactly toward or exactly away from any star—that is, if the star be situated exactly in the apex or the antapex, the observer's motion will contribute nothing to the observed proper motion. The star's parallactic displacement will be zero, in other words, and the diagram of motions for small areas containing the apex and antapex will still be that of Fig. 2. For a group of stars seen in a direction perpendicular to the observer's motion, on the other hand, the parallactic displacements will have their maximum values, and the oval of Fig. 4 will show the greatest degree of elongation. For groups intermediately situated, the diagrams of motion will be ovals, but less elongated.

In the absence of systematic motion, therefore, the diagrams for all parts of the sky, in general, will be ovals of varying degrees of elongation. The major axes, if produced indefinitely, will be great circles passing through apex and antapex. Further, a point of importance, the diagrams will all be symmetrical with respect to their major axes; and, similarly, any other condition of symmetry must also be satisfied. For example, the sums or averages of all the proper motions on the two sides of the major axis will be equal.

Upon constructing the diagrams for the observed motions in 28 different regions of the sky, Kapteyn found that the resulting figures were far from symmetrical. More-

over, a marked regularity in the deviations from symmetry was at once noted. Regions near the North Pole of the galaxy all deviated in one sense; those near the South Pole, in the opposite sense. This was clearly seen from the diagrams themselves and also from the average values of the proper motions on the two sides of the great circle passing through the antapex and the center of each diagram. Thus the average proper motions per century parallel to these great circles for regions in different galactic latitudes are as shown in the table.

| Gal. Lat. | Average P. M. |       | Difference |
|-----------|---------------|-------|------------|
|           | Upper         | Lower |            |
| —65°      | 5".5          | 1".3  | +4.2       |
| —30       | 4.7           | 2.5   | +2.2       |
| —10       | 3.1           | 1.3   | +1.8       |
| +10       | 2.3           | 2.3   | 0.0        |
| +30       | 2.4           | 4.8   | —2.4       |
| +65       | 2.2           | 7.0   | —4.8       |

The difference between the motions in the upper and lower parts of the diagrams are of the same general order as the motions themselves, and the progression in these differences, showing a reversal of the symmetry in passing from pole to pole, is pronounced. The assumption of a simple random distribution of stellar motions was clearly untenable.

By way of elucidation Kapteyn first examined the question as to whether the phenomenon might be due to local motions affecting all the stars in any given region in common, but found a definitely negative answer. In the next step he was guided by the fact that, to some extent at least, the motions do show the characteristics of a random distribution, as though such a distribution were really present underneath superimposed irregularities of unknown origin. The question then naturally presented itself, What form would these irregularities take, if the underlying effects of a random distribution were removed?

Considerations of symmetry show at once that for a random distribution the diagrams of motion for all regions at the same distance from the antapex (or apex) must have the same form. Differences between *observed* diagrams within such a zone of regions must therefore be due solely to the

irregularities of unknown origin (excluding of course observational errors). Differences between diagrams in different zones could not be so interpreted, because the diagram for random distribution itself changes from zone to zone, owing to the varying degree of elongation referred to above. Kapteyn therefore formed a mean diagram for each of several zones by averaging all the results in each zone. Since the symmetry of the original diagrams reverses in passing from northern to southern galactic latitudes and since the latitude of the antapex itself is small, it follows that such a procedure must eliminate to a large degree the influence of these irregularities. In fact, the mean diagrams thus found were very nearly symmetrical, with major axes directed nearly toward apex and antapex, and hence represented the random motions assumed to underlie those actually observed. The contribution of the unknown irregularities was then easily brought to light by forming for each zone the differences between the individual diagrams and the mean for the zone.

The irregularities at once revealed themselves as an excess of proper motions in certain definite directions, most of the 28 regions showing an excess in *two* directions. When plotted on a globe it was seen that each of the two sets of directions converged separately in a point, to an approximation comparable with the uncertainty of the data. The two convergents thus found, known as the apparent vertices of motion, are separated by about  $140^\circ$ , one lying near  $\alpha$  *Orionis*, the other near  $\eta$  *Sagittarii*.

The direction of proper motions toward a convergent point can mean but two things—a common or group motion of the stars along parallel paths toward the convergent, or a recessional motion of the observer away from that point. The latter possibility is excluded because the motion of the observer relative to the center of mass of the whole system is away from the antapex, a point coinciding with neither of the vertices of motion revealed by Kapteyn's analysis.

The existence of systematic motions among the stars was thus demonstrated, and, more than that, the actual dis-

tribution of velocities in space seemed to be clearly indicated. Obviously, two interpenetrating streams of stars in relative motion would give rise to apparent systematic motions having the general character of those actually observed. The true convergents or vertices of two such streams would necessarily be separated by exactly  $180^\circ$ , and would so appear to an observer at rest with respect to the center of mass of the two streams. For an observer in motion, however, the convergent points would not be separated by  $180^\circ$ . Allowance for the known velocity of the solar system showed that the true vertices of the observed stream motions are antipodal points lying exactly in the Milky Way.

The hypothesis of two interpenetrating streams of stars seemed therefore to be justified, but Kapteyn did not at once announce his results. In fact, they were withheld three years, until the International Scientific Congress in St. Louis in 1904. The evidence afforded by his study of proper motions seemed to be conclusive, but he saw that a similar study of radial velocities would yield a crucial test of the theory. This test he was anxious to apply before publication, but the spectroscopic data then available were not well adapted to the purpose and, moreover, were too meager for a detailed analysis. They showed an excess in the average radial velocity for stars near the vertices, as would be expected, but little more. As additional data were not immediately forthcoming, Kapteyn finally announced his conclusions, but it was with some hesitation that he did so. The confirmation to be expected from radial velocities has since been supplied by others.

For the sake of completeness, it should be stated that Kapteyn's division of the stars into separate streams, thus giving rise to a dualistic conception of the group studied, was not essential as an interpretation, as was subsequently shown by Schwarzschild who proposed the ellipsoidal hypothesis. This conception regards the collection of stars as a unit, individual stars moving with respect to the center of mass of the system, but with a relatively high degree of

mobility along a line coinciding in direction with that joining the true vertices of motion found by Kapteyn. As a physical explanation these two viewpoints are quite different, as are the mathematical formulæ to which they lead as means of discussing the observational data; but within the limits of error, both were capable of representing the observed motions of the stars. At a later time Kapteyn showed that the evidence seemed to favor the two-stream hypothesis, rather than the ellipsoidal conception of Schwarzschild; but the decision can scarcely be said to be final, and even now the words of Dyson are not without force<sup>3</sup>.

The dual character of Kapteyn's system should not be unduly emphasized. Division of the stars into two groups was incidental to the analysis employed, but the essential result was the increase in the peculiar velocities of stars towards one special direction and its opposite. It is this same feature, and not the spheroidal character of the distribution, which is the essential of Schwarzschild's representation.

For twenty years Kapteyn gave much thought to star-streaming, and important papers dealing with various aspects of the phenomenon appeared from time to time. Only the last of these,<sup>4</sup> published but a few weeks before his death, can be considered here. The contribution, remarkable for its daring and boldness of conception as well as for the skilful way in which he handled his material, goes far behind the superficial phenomena of the star streams and formulates a tentative theory of the dynamical organization of the stellar universe, and incidentally provides an explanation of the peculiarities of stellar motion which, in a general way, is both qualitatively and quantitatively satisfactory. It is based on, and in fact, is a direct outcome of the results of many years' study of the distribution of stars in space. These results must be borne in mind in order to understand Kapteyn's argument.

Since the days of the elder Herschel it has been believed that the universe of stars we see about us is a limited collection, scattered throughout a watch-shaped region of space with the solar system somewhere near its center. The

<sup>3</sup>*Nature*, **82**, 13, 1910.

<sup>4</sup>*Mt. Wilson Contr.*, No. 230; *Astrophysical Journal*, **55**, 302, 1922.

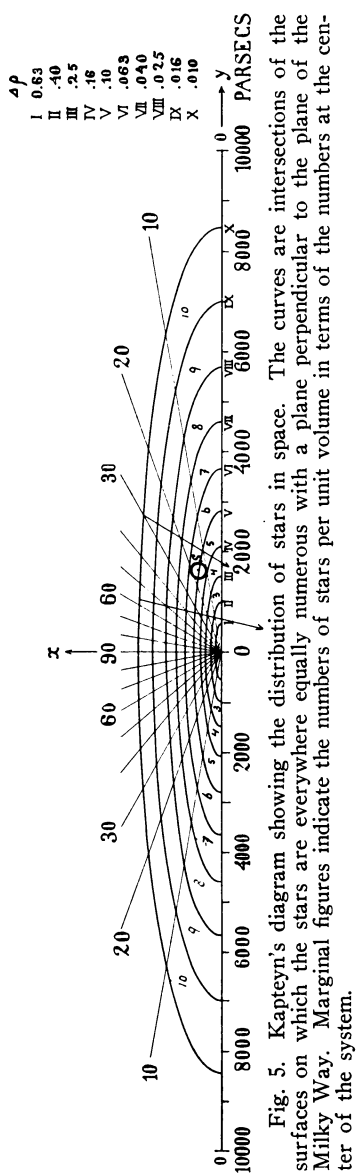


Fig. 5. Kapteyn's diagram showing the distribution of stars in space. The curves are intersections of the surfaces on which the stars are everywhere equally numerous with a plane perpendicular to the plane of the Milky Way. Marginal figures indicate the numbers of stars per unit volume in terms of the numbers at the center of the system.

well-known increase in the number of stars per square degree on the face of the sky as the Milky Way is approached from either side made this explanation plausible at least. It was thought, too, that the numbers of stars per unit of volume—the volume density—must decrease gradually from the center of the collection outward toward its boundary. Kapteyn's study of the magnitudes and parallaxes of individual stars, and the mean parallaxes of stars of different degrees of brightness have given precision to these ideas, and important papers<sup>5</sup> published in 1920 and 1922 contain collections of numerical data for the volume density at different distances and in different directions from the center of the system. Fig. 5 illustrates these results by showing a section through the center perpendicular to the plane of the Milky Way, or rather the half-section lying above this plane. The curved lines are the intersections of surfaces of equal density, the values of the densities themselves being indicated in the margin. For example, the

<sup>5</sup>Mt. Wilson Contr., Nos. 188, 229; *Astrophysical Journal*, **52**, 23, 1920; **55**, 242, 1922.

density of the outermost surface shown is one one-hundredth that at the center of the system. The scale of distances, in parsecs ( $1 \text{ parsec} = 3.26 \text{ light years}$ ) may be read from the bottom of the diagram. The observational data show that the equi-density surfaces are very nearly ellipsoids of revolution, the approximation being so close that for his investigation of the dynamics of the system Kapteyn assumed them to be actually of this form.

There are two fundamental assumptions underlying the investigation, neither of which is open to any direct test of validity. The first is that the system is in a steady state with respect to the forces of attraction exerted by the system as a whole upon the individual stars. Together with the form of the equi-density surfaces, this means that systematic rotational motion about the shortest axes of the density ellipsoids must exist. Were there no such rotation, the assumption of a steady state would imply equi-density surfaces of spherical form. The rotation about the axis need not, however, be in the same direction for all the stars, and Kapteyn therefore supposes that rotation takes place in both directions.

There are very good reasons for believing that the solar system is not at the geometrical center of the system, but distant therefrom several hundred parsecs, perhaps a thousand parsecs, even. Further, the volume of space surrounding the observer that contains the stars for which he has thus far been able to detect appreciable proper motions is relatively small. The systematic rotations of the stars about the axis perpendicular to the plane of the Milky Way, some in one direction, some in the opposite direction, will therefore present themselves in the observed proper motions as the relative drift of two interpenetrating streams, which, under the conditions stated, will give little evidence of curvature of path. Moreover, the stream motion will be parallel to the plane of the Milky Way, and, since the solar system is not far from this plane, the true vertices of the streams will lie very nearly in the Milky Way itself.

This, briefly, is Kapteyn's final interpretation of the phe-

nomena of star streaming. Obviously, rotational motions of the kind required by the fundamental assumption of a steady state must produce systematic effects in the proper motions qualitatively like those actually observed. But Kapteyn goes much farther and derives certain numerical results which, in part at least, are known to be of the right order of magnitude. First of all, the true vertices of the streams, which are in longitudes  $167^\circ$  and  $347^\circ$ , respectively, must be  $90^\circ$  from the geometrical center of the system. Hence, the center of the system must have a longitude of either  $77^\circ$  or  $257^\circ$ . The evidence for a choice between these two values is not very convincing. Investigations by Herschel, Struve, and Kapteyn himself are in good accordance with  $77^\circ$ ; but studies of the density distribution of various classes of objects suggest the antipodal point as the direction of the center. It is perhaps noteworthy that Strömberg<sup>6</sup> from a preliminary discussion of radial velocities found evidence of curvature in the streams with the center of motion in the direction of  $257^\circ$ .

For the remainder of his discussion Kapteyn makes use of the second of the two unverifiable assumptions referred to above, namely, that in the vicinity of the axis of rotation, where the influence of rotational motion is negligibly small, the stars behave as do the molecules of a gas in a quiescent atmosphere. In other words, the stars have peculiar velocities distributed in accordance with Maxwell's law, but no systematic motion. It was well known to Kapteyn that Maxwell's law could not be strictly applicable because the observed proper motions and radial velocities both show an excess of large values over those to be expected on the basis of this law. Nevertheless, the deviations are not such as to prevent its use in a first approximation, provided the behavior of objects as different in size as stars and molecules is in other respects comparable. The latter is the essential feature of Kapteyn's assumption, which he made with much hesitation, although necessarily it then seemed, as a means of further advance.

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<sup>6</sup>*Mt. Wilson Contr.*, No. 144; *Astrophysical Journal*, 47, 7, 1918.

He was thus led to an equation analogous to that used for barometric determinations of altitude in an atmosphere of constant temperature, which for any given point on the axis involves the average peculiar velocity of the stars, the stellar density, the distance from the center of the system, the attraction of the whole system exerted at the point, and, finally, the acceleration produced by a star of average mass. This equation is readily transformed into an expression involving the distance, the attraction, and the average stellar mass expressed in that of the Sun as a unit. The attraction can be calculated from the law of density distribution expressed graphically in Fig. 5. Hence, values of the average mass of the stars at various points along the axis of rotation can be found. These range from 2.2 times the solar mass at 200 parsecs to 1.4 at 1660 parsecs. Although in excess of the probable average, which is about 0.8 the mass of the Sun, these quantities are of the right general order of magnitude. As Kapteyn remarks, however, the averages found represent the total mass, including non-luminous material, divided by the number of luminous stars, and must therefore be too large if there is an appreciable amount of dark material scattered through space.

An equation similar to that for points on the axis of rotation can also be established for points in the plane of the Milky Way, except that in the latter case a term depending on the acceleration produced by the rotation of the system, and hence on the angular velocity, must also be included. Using the values of the average mass just found, Kapteyn was thus able to calculate the angular velocity itself. For distances in excess of 2000 parsecs this he found to be nearly constant and equal to 19.5 km per second. Since part of the stars are supposed to rotate in one direction and the remainder in the opposite direction, it follows that the linear motion of one stream relative to the other would be  $2 \times 19.5 = 39$  km per second. The agreement with the directly observed motion derived from radial velocities, for which Kapteyn adopts 40 km per second, is very close. Finally, from the distribution of stellar density and from cer-

tain details connected with the rotation he is led to assign tentatively a value of 650 parsecs for the distance of the solar system from the geometrical center of the stellar universe.

In brief, therefore, the results of Kapteyn's investigations of the form and structure of the stellar universe, supplemented by the assumption of a dynamically steady state, lead him to a theoretical stream motion parallel to the plane of the Milky Way with vertices in the Milky Way itself. The qualitative characteristics, therefore, are just those of the observed star streams. His second assumption, which assigns to the stars properties analogous to those of the molecules of a gas, affords on the other hand, means of calculating the average mass of the stars and the value of the stream velocity itself. Both the numerical results thus found are of the right order of magnitude.

No one could be more conscious of the doubtful points and the tentative character of these results than Kapteyn himself. Throughout the discussion he carefully enumerates the objections to which his argument is open and the limitations imposed by the defectiveness of the material with which it is supported. A more intimate view of his own attitude is revealed by a personal letter from which I may now quote:

I never sent a paper to press with such mingled feelings as I do the present one. You will understand this in perusing it. If the application of the kinetic theory of gases made therein is correct, then I feel personally confident that my solution for the structure and motion of the great stellar system is right too, and that, notwithstanding the fact it is but a first crude approximation. If I had been a younger man I should probably have kept back my solution for at least a year in order to make further approximations. As it is, these seemed to me too dangerous, especially as I know so little about the kinetic theory. I therefore publish at once with the feeling that the probability of the theory must certainly be sufficient for any good mathematician or mathematical physicist to try for a more rigorous solution. For it is now a purely mathematical question to make out whether the theory is right or not, and this being so, I am confident that it will not be long before we know with certainty. If it is right, it is just the completion of my life work I had always hoped for.

As for the application of the kinetic theory, I consulted many experts. Opinions are divided. Some think that in the main it must surely be right; others are very doubtful. I am sorry to say that just those I trust most are more or less doubtful. I certainly therefore do not look forward to any general acceptance of the theory, but I have my hope

that the paper may occasion a good deal of discussion, at the end of which we may come to an improved reliable solution.

Kapteyn's theory of the arrangement and motion of the sidereal system was communicated at the Edinburgh meeting of the British Association for the Advancement of Science in September, 1921, and proved, as he hoped it would, a stimulus to further theoretical discussion. Thus Jeans<sup>7</sup> has pointed out that the assumption of a steady state is probably the most reasonable that can be taken as a basis for a dynamical discussion and has made an important contribution by showing that if the observed stream velocity of 40 km per second be adopted as a known quantity, all Kapteyn's other results can be derived from the assumption of a steady state alone. With this slight limitation, the introduction of the kinetic theory is avoided altogether. Jeans thus finds for the average stellar mass  $4.2 \times 10^{33}$  grams or 2.5 times the solar mass, and 700 parsecs, an admittedly rough approximation, for the distance from the Sun to the center of the system. A further gain is that Jeans is able to calculate the average peculiar velocities of the stars at different points in the system, whereas Kapteyn used an average value derived from observations, which he assumed to be constant throughout the system.

Jeans's discussion is therefore of the greatest importance in increasing the probability of Kapteyn's dynamical theory, and affords much of the confirmation that Kapteyn himself so greatly desired. And it is of interest, too, in that indirectly it contributes a certain amount of justification to Kapteyn's assumption as to the applicability of the kinetic theory to the study of stellar phenomena. But neither Kapteyn nor Jeans would maintain that anything more than a beginning has been made in the study of a very complex problem whose intricacies will become more apparent as more extensive data are accumulated. Something of this is already foreshadowed by Strömberg's<sup>8</sup> recent investigation of late-type stars, which indicates that the dwarf stars are to be identified with Kapteyn's first stream, while the giants

<sup>7</sup>*Monthly Notices*, **82**, 122, 1922.

<sup>8</sup>*Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 1922.

constitute the second stream, both the streams showing an internal distribution of velocities of the ellipsoidal type.

The most pressing need for further study of the structure of the stellar universe is still the accumulation of observational data. All his life Kapteyn emphasized the importance of this very practical consideration, and until the end it was his great concern that there should be no cessation of activity in gathering the facts essential to further advance. And it is no small part of his contribution to science that in his "Plan of Selected Areas" he outlined with such clearness the way to be followed. The care with which he pointed out what was essential recalls the words of Mr. Elihu Root:

Occasionally a man appears who has the instinct to reject the negligible. A very great mind goes directly to the decisive fact, the determining symptom, and can afford not to burden itself with a great mass of unimportant facts.

When Kapteyn began his work there was little to suggest that the questions in which he was so much interested stood in any close relation to those of the physical constitution and development of stars; but as the years passed it appeared that the form and structure of the universe cannot be separated from the physical characteristics of the units that compose it. This led naturally to his visits to the Mount Wilson Observatory, which was established primarily to study stellar evolution, and to his connection with that institution as one of its Research Associates. This association was severed only by his death, although he had not been in California since 1914. The Mount Wilson Observatory owes much to his vast experience and wise counsel.

Kapteyn's investigations early gained the high appreciation of his colleagues. He has been signally honored by the universities and learned societies of other lands, as well as by those of his own. The bestowal of the Bruce Medal by this Society in 1913 is, by itself, sufficient to mark him as a commanding figure in astronomical science. Although the ultimate importance of no man's work can be estimated justly at the moment his labors cease, it is clear that none

since Sir William Herschel has made such important contributions to our knowledge of the form and structure of the stellar universe as Kapteyn; and if the successful stimulation of others to fruitful effort is to be reckoned an achievement worth while, then for this, too, Kapteyn's name must ever stand high, for in this respect, also, his accomplishment has been remarkable.

Mount Wilson Observatory, August, 1922.